金證照

Credit risk
FRM P2

AIM 43-1
Describe the Merton model for corporate security pricing, including its assumptions, strengths and weaknesses

- In the original option pricing paper, Black and Scholes suggested that their methodology could be used to price corporate securities. Merton used their intuition and applied it to price corporate security. According to Merton, the payoff to equity holders (security price) is similar to the payoff of a call on the value of the firm struck at K. Merton makes the same assumptions as Black and Scholes, and the call can be priced using option prices derived as:
- where N(.) denotes the cumulative normal distribution and r the riskless interest rates.

$$S_t = V_t N\left(k + \sigma_V \sqrt{T-t}\right) - Ke^{-r(T-t)} N(k)$$

$$k = \frac{\log(V_t / K) + \left(r - \frac{1}{2}\sigma_V^2\right)(T-t)}{\sigma_A \sqrt{T-t}}$$

The Merton model brings a lot of insight into the relationship between the fundamental value of a firm and its securities. The original model relies on very strong assumptions:

- ❑ The capital structure is simplistic: equity + one issue of zero-coupon debt.
- ❑ The value of the firm is assumed to be perfectly observable.
- ❑ The value of the firm follows a lognormal diffusion process. With the type of process, a sudden surprise (a jump), leading to an unexpected default, cannot be captured.
- ❑ Default can only occur at the debt maturity.
- ❑ Riskless interest rates are constant through time and maturity.
- ❑ The model does not allow for debt renegotiation between equity and debt holders.
- ❑ There is no liquidity adjustment.

These stringent assumptions may explain why the simple version of the Merton model can not well cope with empirical facts observed on the market. Practical difficulties also play a part in hampering the empirical relevance of the Merton model:

- ❑ The value of the firm is difficult to find because the marked-to-market value of debt is often unknown.
- ❑ All that relates to goodwill or to off-balance-sheet elements is difficult to measure.
- ❑ The estimation of assets volatility is difficult due to the low frequency of observations.

Example :

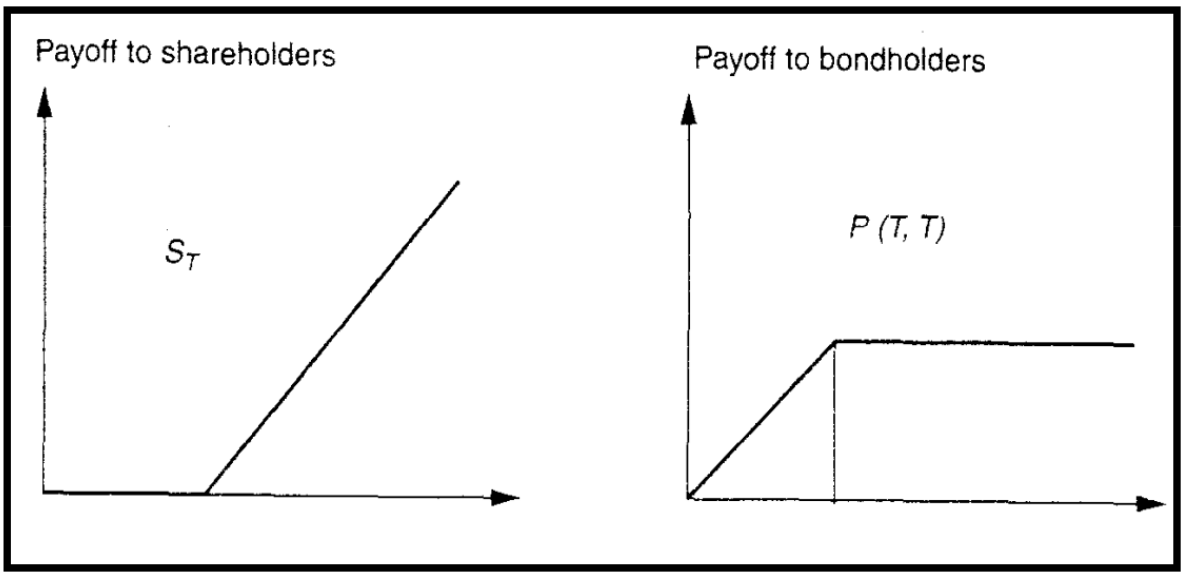
- Calculate the value of the firm's equity at T, ST, given that principal amount due on the zero-coupon bond is 60 million and the total value of the firm at T, VTis $80 million. In addition, what is the value of equity if VTis $50 million?
- ST= Max(80 - 60, 0) = $20 million
- ST= Max(50 - 60, 0) = $0 million

Example :

- Calculate the value of the firm's debt at T, DT, given that principal amount due on the zero-coupon bond is 60 million and the total value of the firm at T, VTis $50 million. In addition, what is the value of debt if VTis $80 million?
- DT= 60 - Max(60 - 50, 0) = 60 - 10 = $50 million
- DT= 60 - Max(60 - 80, 0) = 60 - 0 = $60 million

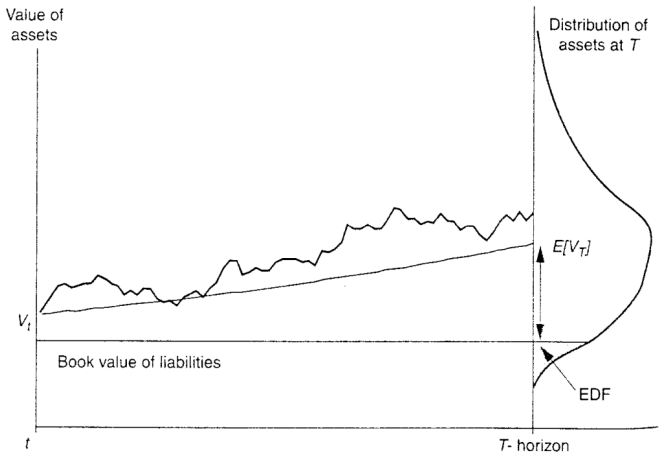
$$\text{payment to debtholders} = D_M - \max(D_M - V_M, 0)$$

$$\text{payment to stockholders} = \max(V_M - D_M, 0)$$



AIM 43-2
Describe the Moody's KMV Credit Monitor Model to estimate probability of default using equity prices, and compare the Moody's KMV equity model with the Merton model

- KMV Credit Monitor applies the structure approach to exacting probabilities of default at a given horizon from equity prices. Equity prices are available for a large number of corporates. If the capital structure of these firms are known, then it is possible to exact market-implies PD from their equity price. The probability of default is called **expected default frequency (EDF)** by KMV. In KMV model, EDF is derived from **Distance to Default(DD)**



Comparison of Merton and KMV:

- ❑ Merton model only applies to firms financed by equity and one issue of zero-coupon debt. KMV model assumes that the capital structure of an issuer consists of long debt (LT) and short debt (ST). The default point, the asset value at which the firm will default, generally lies somewhere between total liabilities and short debt, i.e.,
- Default value X = ST + 0.5 LT If LT/ST < 1.5
- Default value X = ST + (0.7 – 0.3 ST/LT)/LT otherwise
- The rule of thumb above is purely empirical and does not rest on any solid theoretical foundation.
- ❑ The estimation of the firm value process is also difficulty by Merton model, since the value of the firm asset is unobservable. KMV model uses a proprietary undisclosed methodology to calculate the volatility.
- ❑ In Merton model, the probability of default is

$$PD_t = N\left(-\frac{\left[\log(V_t) - \log(X) + \left(\mu - \sigma_V^2/2\right)(T-t)\right]}{\sigma_A\sqrt{T-t}}\right)$$

- The EDF of KMV model takes a very similar form. It is determined by a distance to default
- Unlike Merton's model, KMV does not rely on the cumulative normal distribution N(.). Default probabilities calculated as N(-DD) would tend to be much too low due to the assumption of normality (too thin tails).

- Distance to default(DD)

$$DD = \frac{\text{expected asset return} - \text{default threshold}}{\sigma_{\text{expected asset returns}}}$$

- Expected default frequency (EDF) in KMV is also the probability of default
- Example:
- An asset's expected return is 700 and default threshold is 500, its annual standard deviation is 100:
- Then DD=(700-500)/100=2
- Assume 1000 firms last year had a DD of 2 and 15 of these firms defaulted after a year. EDF=15/1000=1.5% should be assigned to rating BB

	1 year Transition Matrix							
From Rating	To Rating							
	AAA	AA	A	BBB	BB	B	CCC	D
AAA	88,658%	10,294%	1,017%	0,000%	0,031%	0,000%	0,000%	0,000%
AA	1,079%	88,705%	9,553%	0,342%	0,145%	0,145%	0,000%	0,031%
A	0,063%	2,876%	90,205%	5,919%	0,740%	0,177%	0,010%	0,010%
BBB	0,053%	0,339%	7,069%	85,238%	6,053%	1,005%	0,085%	0,159%
BB	0,033%	0,077%	0,557%	5,680%	83,572%	8,083%	0,535%	1,464%
B	0,011%	0,044%	0,174%	0,652%	6,595%	82,703%	2,760%	7,062%
CCC	0,000%	0,000%	0,660%	1,050%	3,050%	6,110%	62,970%	26,160%
D	0,000%	0,000%	0,000%	0,000%	0,000%	0,000%	0,000%	100,000%

AIM 43-3
Describe credit scoring models and the requisite qualities of accuracy, parsimony, non-triviality, feasibility, transparency and interpretability

- Credit scoring models are used to determine small and private companies' default probability.
- Accuracy: Having low error rates arising from the assumptions in the model.
- Parsimony: Not using too large a number of explanatory variables.
- Non -triviality: Producing interesting results.
- Feasibility : Running in a reasonable amount of time and using realistic resources.
- Transparency and interpretability: Providing high-level insight into the data relationships and trends and understanding where the output of the model comes from.

AIM 43-4
Define and differentiate among the following quantitative methodologies for credit analysis and scoring

- Linear discriminant analysis

The basic principal aim of discriminant analysis is to segregate and classify a heterogeneous population in homogeneous subsets. Looking for a linear combination of explanatory variables which separates (maximum distance) the two classes most.

- Parametric discrimination

solves the problem is to apply a transformation to the score in order to obtain a probability, which is bounded into 0 and 1. In most cases the choice will not be as simple as being either in class 1 or class 2 , and one would need probabilities of being in a given class. A very crude approach would be to apply a simple linear regression. While the transformation process may also be non-linear, dependent on whether the variable is monotonic or non-monotonic.